Modeling transient strain-rate partitioning during porphyroblast growth

W.G. Groome* and S.E. Johnson
Department of Earth Sciences, University of Maine, Orono, ME, 04469, USA
*wesley.groome@umit.maine.edu

Overview

Changing mineralogy during metamorphism can significantly affect the strength of mid-crustal rocks, and by extension the strength of relatively large volumes of the middle crust. The growth of relatively strong porphyroblasts during prograde metamorphism is a common phenomenon in metapelitic rocks and the increasing abundance of effectively rigid porphyroblasts in rocks of these bulk compositions can markedly increase their strength relative to surrounding rocks. The localization of metamorphic strengthening reactions in a stratigraphic succession leads to changes in the strain-rate partitioning within the layer undergoing the reaction and the stratigraphic succession as a whole. Changes in strain-rate partitioning during porphyroblast growth can cause further metamorphic reactions in high strain-rate zones, which can in turn lead to additional strain-rate partitioning, causing a feedback between metamorphic reaction and strain-rate partitioning (e.g. Bell and Hayward, 1991, Bell et al. 2004). This feedback can cause strain-rate partitioning on the orogen scale, which can in turn cause metamorphism in other parts of the crust (e.g. Bell et al., 2004), affect the exhumation of high-grade metamorphic rocks (e.g. Jamieson et al., 2002), and potentially alter the topography of an orogen. In this contribution, we explore the evolving strain-rate
partitioning behavior within a layered succession during porphyroblast growth, and explore the feedback between strain-rate partitioning around effectively rigid porphyroblasts and metamorphic reactions.

**Theoretical Background and Natural Example**

The strength of a polymineralic rock is largely a function of the strength, volume fraction and distribution of the constituitive minerals (e.g. Burg and Wilson, 1987; Jordan, 1988; Handy, 1990; Tullis et al. 1991; Ji and Zhao, 1993; Bons and Urai, 1994; Goodwin and Tikoff, 2001; Ji and Xia, 2002; Treagus, 2002; Johnson et al. 2004). The growth of large, effectively rigid grains (porphyroblasts) during prograde metamorphism can cause pelitic layers to strengthen relative to unmetamorphosed pelitic layers and potentially relative to interlayered lithologies that have not experienced porphyroblast growth (e.g. Groome and Johnson, in review). If porphyroblast abundance is high enough in the pelitic layer, it may become stronger than interlayerd units and lead to an overall strengthening of large crustal volumes.

In unmetamorphosed turbidite sequences, psammite layers consist predominantly of quartz and feldspar, and pelite layers consist predominantly of phyllosilicates, quartz and feldspar. The predominance of relatively weak phyllosilicates in unmetamorphosed pelite units makes them weaker than interlayered psammite units during layer-parallel shearing at low to intermediate temperatures (e.g. Shea and Kronenberg, 1993; Treagus, 1999). However, during prograde metamorphism of pelitic rocks, the relative abundance
of phyllosilicates can decrease due to the increased abundance of effectively rigid porphyroblasts, which leads to changes in the relative strengths of pelitic and psammitic rocks.

The White Mountains region in the northern New England Appalachians records evidence for the strengthening of pelitic layers relative to psammitic layers during prograde metamorphism. During prograde metamorphism and deformation, large andalusite grains grew in the pelitic layers, making them stronger than the psammitic layers. Andalusite grains are up to 25 cm long and constitute up to 25-30% by volume of the pelitic layers (e.g. Wall, 1988; Eusden et al. 1996). By using the foliation refraction technique of Treagus (1999) to estimate effective viscosity contrasts between adjacent rock types, we determined that the pelitic layers were approximately 1.5-2.0 times more viscous than the interlayered porphyroblast-free psammitic layers (Groome and Johnson, in review). The use of foliation refraction angles to estimate effective viscosity contrasts is based on the relationship:

\[
\frac{\tan \theta_a}{\tan \theta_b} = \frac{\eta_a}{\eta_b} \tag{1}
\]

where subscripts \(a\) and \(b\) refer to two distinct lithologies, \(\theta\) is the angle between bedding and foliation in a given bed and \(\eta\) is the effective viscosity of a given bed (Treagus, 1999). This method allows for the estimation of relative viscosity contrasts between contrasting lithologies, but does not allow for the estimation of absolute viscosity values.

Using our field example from the New England Appalachians as a constraint on the strength contrast between pelitic and psammitic bulk compositions in amphibolite-
facies metaturbidites, we present a model of the evolving viscosity contrast between layers undergoing porphyroblast growth and layers without porphyroblasts. This changing viscosity contrast leads to changes in strain-rate partitioning between model pelitic layers and model psammitic layers as the pelitic layers strengthen during porphyroblast growth. Our models consist of a two layer system with contrasting viscosities (Figure 2). The lower layer has a viscosity 2.5 times that of the upper layer, which is consistent with estimated viscosity contrasts between unmetamorphosed pelitic and psammitic bulk compositions (e.g. Treagus, 1999). Isolated, fine grains within the model pelite layer were selected to be porphyroblasts, with viscosities two orders of magnitude greater than the surrounding matrix, making the porphyroblasts effectively rigid. These porphyroblasts were allowed to grow, resulting in an increase in the volume fraction of rigid porphyroblasts with each time step, which caused the viscosity structure of the model to change.

**Implementation in Elle**

In the experiment presented here, we use the following routines in sequence: 1) finite element deformation in *Basil*, 2) grain growth using *elle_gg*, and 3) porphyroblast growth using *elle_pblast*. The implementation of each process is briefly summarized below.

*Basil: Viscous Deformation*
Basil is a two-dimensional viscous finite element code that can solve for linear and non-linear rheologies by the constitutive relationship (Chapter 4.8):

\[ \tau_{ij} = 2 \left[ \frac{B}{\dot{E}} \right]^{\frac{1-n}{n}} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \]  

(2)

where \( \tau_{ij} \) is the yield stress, \( B \) is a material constant in the Elle input file analogous to viscosity, \( \dot{E} \) is the second invariant of the strain rate tensor, \( n \) is a stress exponent that can vary from 1 to 3, and \( u \) and \( v \) are the velocity components in the \( x \) and \( y \) directions, respectively. In our experiments we use a linear viscous rheology \( (n=1) \) such that Equation (2) reduces to:

\[ \tau_{ij} = B \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \]  

(3)

The experiments presented here use a plane strain approximation with cyclic boundaries to simulate simple shear deformation (see Chapter 4.8 for full discussion of Basil implementation). The models discussed below were deformed a very small increment in Basil (one deformation time step) in order to examine the evolving viscosity structure of the layered sequence and the corresponding evolving shear-strain-rate partitioning. Two models were deformed to shear strains of approximately 1.5 (a porphyroblast-free model and a model with a porphyroblast volume fraction of approximately 18%) to illustrate the difference in strain-rate partitioning as porphyroblast abundance increased in one layer.

\textit{Elle\_gg: Grain Growth}

The grain growth routine used for the experiments presented here calculates the grain boundary energy of each grain based on the grain boundary curvature such that high
angle grain boundaries have high grain boundary energies (See Chapter ???.) The driving force for grain growth is the energy required to minimize the grain boundary curvature. The lowest energy configuration in *Elle* is grains with 120 degree triple junctions. The rate of grain growth is governed by the equation:

\[ R = EM \]  

where \( R \) is the rate of grain growth, \( E \) is the driving energy (proportional to the curvature of the grain boundary) and \( M \) is the grain boundary mobility.

*Elle_pblast: Porphyroblast Growth*

The porphyroblast growth routine used in these experiments allows selected grains to grow relative to the surrounding grains. The growth rate of the porphyroblasts is fast relative to surrounding grains and is not governed by surface energy driving forces, as in *Elle_gg*. The number of porphyroblast growth stages per time step is adjustable in the input code such that porphyroblasts can grow at varying rates relative to the rate of deformation. The growing porphyroblasts obey the same governing equations as in the grain growth routine, and will attempt to form circular grains as they get larger. In these experiments, the porphyroblasts are assigned a viscosity two orders of magnitude higher than the surrounding grains, making them effectively rigid.
Experiments and Discussion

Description of the Model

The experiments shown here have porphyroblasts growing in the pelitic layer without deformation, and are referred to as static growth models. Porphyroblast abundance in the pelitic layer ranges from 0% in the initial geometry to ~18% in the final geometry. In addition, the two end-member geometries were deformed in simple shear to shear strains of approximately 1.5 to illustrate the change in strain partitioning as porphyroblast abundance increases. Figure 2 shows the geometry used at four stages of porphyroblast abundance (0.5%, 3%, 4.5% and 10% porphyroblast in the pelitic layer). The pelite layer has an assigned dimensionless viscosity of 1.0 ($B$ in Equation (3)) and the psammite layer has an assigned dimensionless viscosity of 2.5, giving a psammite to pelite viscosity ratio of 2.5, which is consistent with estimated viscosity contrasts using field data (e.g. Treagus, 1999). The porphyroblasts in the model have an assigned dimensionless viscosity of 100, making them effectively rigid objects in the viscous model. Porphyroblasts in the initial stage were placed in the matrix by hand and were distributed through the pelite layer. Porphyroblast abundance at each stage was estimated by dividing the area covered by porphyroblast by the total area of the pelitic layer alone (i.e. the psammitic layer was not included in the calculation of porphyroblast abundance).

The viscosity of the system was calculated using a Gaussian integration across the $x$-dimension of the model to obtain an integrated viscosity for each $y$-value in the
geometry. The bulk viscosity of the pelitic layer was calculated by integrating the area under the viscosity profile from the bedding contact to the top of the model. A single Basil deformation step was conducted for each model to obtain a shear-strain-rate contour map for each stage of porphyroblast growth (via Equation (1)), which provides the basis for the viscosity calculation.

**Results**

Figure 3 shows the evolving viscosity structure and strain-rate partitioning in our layered system as the porphyroblast abundance increases in the pelitic layer. The starting geometry, with no porphyroblasts, has a uniform viscosity structure through the pelitic layer, and strain rate is uniformly partitioned into the pelitic layer. As porphyroblast growth occurs, however, the viscosity structure and strain-rate partitioning through the pelitic layer becomes less regular. High viscosity regions correspond to areas that have high porphyroblast abundance, whereas low viscosity regions correspond to porphyroblast-free areas. As porphyroblast abundance increases, the shear-strain-rate partitions around porphyroblasts within the pelitic layer, in addition to larger-scale partitioning into porphyroblast-free psammitic layers. Figure 4 shows the evolving viscosity structure of just the pelitic layer as porphyroblast abundance increases from 0.5% to 18%. High viscosity spikes in the viscosity structure correspond to areas in the geometry where porphyroblast abundance is high, whereas low viscosity spikes correspond to areas in the geometry where porphyroblast abundance is low. The low viscosity spikes in the pelitic layer correspond with zones high shear-strain-rate.
The changing bulk viscosity of the model pelite layer as porphyroblast abundance increases is shown in Figure 5. The trend of increasing viscosity in our model falls between two end-member theoretical bounding conditions describing the strength of poly-phase materials: the Voight Bound and the Reuss Bound. The Voight Bound assumes that the strain rate for each phase (and the bulk strain) is the same and that the bulk strength \( \sigma_c \) of a polyphase material increases linearly with increasing volume fraction \( V_s \) of the strong phase \( \sigma_s \) (e.g. Ji and Xia, 2002):

\[
\sigma_c = V_s \sigma_s + (1 - V_s) \sigma_w \tag{5}
\]

where \( \sigma_w \) is the strength of the weak matrix phase. The Reuss Bound assumes that all phases are subjected to a constant differential stress and that the bulk strain rate \( \dot{\varepsilon}_c \) is a function of the strain rate in each phase \( \dot{\varepsilon}_s, \dot{\varepsilon}_w \) (e.g. Ji and Xia, 2002):

\[
\dot{\varepsilon}_c = V_s \dot{\varepsilon}_s + (1 - V_s) \dot{\varepsilon}_w \tag{6}
\]

Most treatments of the rheology of natural polyphase materials assume that natural rheologic trends fall between the two theoretical end-members (e.g. Ji and Xia, 2002), which is consistent with the experiments presented here.

Two geometries were deformed in simple shear to bulk strains of approximately 1.5 to illustrate the difference between strain partitioning in a porphyroblast-free layered
system and a porphyroblastic layered system. In the absence of porphyroblasts, the pelitic layer in our model has a viscosity 0.4 times that of the psammitic layer. Strain is partitioned in this model such that the pelitic layer records a shear strain of approximately 2.1 and the psammitic layer records a shear strain of approximately 0.8, consistent with the viscosity contrast between the two layers (Figure 6). A second geometry, with approximately 18% porphyroblast in the pelitic layer, was deformed to a similar bulk strain. In this model, strain is not partitioned as significantly between the two layers, with the pelite layer recording a shear strain of approximately 1.6 and the psammite layer recording a shear strain of approximately 1.4 (Figure 6), because with approximately 18% porphyroblast the pelitic layer has a bulk viscosity similar to the psammitic layer (Figure 5). Furthermore, in the pelitic layer, shear-strain-rates are not uniformly distributed through the layer, but instead partition around the porphyroblasts.

Conclusions

Porphyroblast growth during metamorphism will strengthen the rock if the product porphyroblasts are stronger than the reactant phases. Within the layer experiencing porphyroblast growth, the strain rate distribution will become more heterogeneous as porphyroblast abundance increases and strain is partitioned into ever decreasing rock volumes around the porphyroblasts (Figure 6; e.g. Bell and Hayward, 1991). This increasing strain rate partitioning around the porphyroblasts leads to the development of high strain rate gradients along the margins of the porphyroblasts. In nature, this may allow for grain boundaries to dilate along the margins of the
porphyroblasts, which will provide pathways for fluids in the rock that can enhance metamorphic reaction rates (e.g. Etheridge et al., 1983). If the rock is relatively anhydrous, as expected in the lower crust, the opening of grain boundaries will allow for fluids to catalyze retrograde metamorphic reactions, as reported from the Siesa Zone in the Swiss Alps (e.g. Rubie, 1986, Brodie and Rutter, 1985; Koons et al. 1987; Freugh-Green, 1994). However, if the rock is hydrous, as would be expected during the prograde metamorphism of pelitic rocks in the middle crust, the opening of grain boundaries may allow for fluids to leave the immediate reaction site, thus allowing dehydration to progress (e.g. Etheridge et al., 1983).

High strain rates in grains along the margins of porphyroblasts may increase dislocation densities in grains undergoing intracrystalline deformation (e.g. Bell and Hayward, 1991). As dislocation densities increase, the free energy available for metamorphic reactions is increased which will enhance the metamorphic reaction rate if the pressure-temperature conditions are overstepped (e.g. Porter and Easterling, 1992). In nature, this may lead to rapid reaction rates if a reaction is overstepped and porphyroblasts begin to nucleate (e.g. Bell and Hayward, 1991). Once a porphyroblast nucleates and begins to grow, the increasing strain rates in the surrounding grains will add strain energy to the system, which can lower the energy barrier for further progress of metamorphism.

As a rock strengthens during porphyroblast growth, strain rates will be partitioned out of the regions undergoing porphyroblast growth. The partitioning of high strain rates
into surrounding crustal volumes may catalyze metamorphic reactions in parts of the crust that are not undergoing metamorphism (e.g. Bell et al., 2004). If the rocks in these crustal volumes are in disequilibrium with respect to the ambient pressure and temperature conditions, the strain energy added to these rocks as they begin to deform may be enough to overcome the energy barrier for nucleation and growth. If this energy barrier is overcome, metamorphic reactions will occur, potentially strengthening these crustal volumes and leading to further strain rate partitioning into other parts of the crust. In this way a feedback relation between porphyroblast growth and strain-rate partitioning can develop and metamorphic reactions can occur over large crustal volumes in a relatively short period of time (e.g. Bell et al., 2004).

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References


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Figure Captions:

Figure 1: Field photograph from Mt. Washington, New Hampshire, showing refracted foliation at the contact between metapsammitie and metapelite. The bedding-foliation angle in the metapelite layer is larger than the bedding-foliation angle in the metapsammite layer, suggesting that the metapelite had a higher effective viscosity than the metapsammite (see Equation (1)).

Figure 2: Model topologies used in this paper at various porphyroblast abundances (0.5%, 3%, 4.5% and 10%). Porphyroblast abundances were calculated by dividing the area covered by porphyroblast by the area of the pelite layer (i.e. the area of the psammitic layer is not included). The porphyroblasts are the darkest grains in each frame.

Figure 3: Contour maps of shear strain rate and plots showing the viscosity structure and shear strain rate distribution of the model for five stages of porphyroblast growth (0%, 1%, 4.5%, 10% and 18%). Note how the shear strain rate increases in the psammitic layer as porphyroblast abundance increases, corresponding to an increase in the bulk viscosity of the pelitic layer. The Elle topologies for each stage are overlain on the strain-rate maps.

Figure 4: Plot of the evolving viscosity structure of the pelitic layer as porphyroblast abundance increases from 0% to 18%. The location of the starting pelite viscosity and the psammite viscosity are indicated.
Figure 5: Graph of the changing viscosity of the pelitic layer as porphyroblast abundance increases. The theoretical Voight and Reuss bounds are also shown with the viscosity values used in this model as parameters. Note that the trend line for this experiment lies between the two theoretical bounds. All viscosities are normalized to the porphyroblast-free pelite layer. Also shown are the values of the psammite layer and a layer consisting of 100% porphyroblast (Pblast).

Figure 6: Top: Shear strain rate contour maps for the porphyroblast-free simple shear deformation experiment. Note the homogeneous shear strain rate in each layer and the bulk strain rate partitioning into the pelitic layer. Middle: Shear strain rate contour maps for the 18% porphyroblast simple shear deformation experiment. Note strain rate partitioning within the pelitic layer with high strain rate zones around the porphyroblasts. Also, the strain rate is higher in the psammitic layer than in the porphyroblast-free experiment. Bottom: Close-up images of the shear strain rate distribution around porphyroblasts showing high and low shear strain rate zones. The image on the left has had the topology removed for simplification. The location of strain-assisted reaction between two porphyroblasts is indicated. See text for discussion.
Groome and Johnson, Figure 1

Groome and Johnson, Figure 2

Pelite

Psammitic
Groome and Johnson Figure 3

0% Porphyroblast

1.5% Porphyroblast

4.5% Porphyroblast

10% Porphyroblast

18% Porphyroblast
Groome and Johnson Figure 4

0% Pblast

1.5% Pblast

4.5% Pblast

10% Pblast

18% Pblast

Shear Strain Rate

Integrated Viscosity

Pel

Psa

Pel

Psa
Groome and Johnson Figure 5

[Diagram showing normalized viscosity plotted against percentage of porphyroblast, with boundaries labeled Voight and Reuss Bound, and categories of rocks such as Pelite and Psammite.]
Groome and Johnson, Figure 6

Porphyroblast-Free
Shear Strain Rate Contours

Initial Geometry
Deformed Geometry

~18% Porphyroblasts
Shear Strain Rate Contours

Initial Geometry
Deformed Geometry

Strain-Assisted Reactions
High Strain Rate Zone
Low Strain Rate Zone